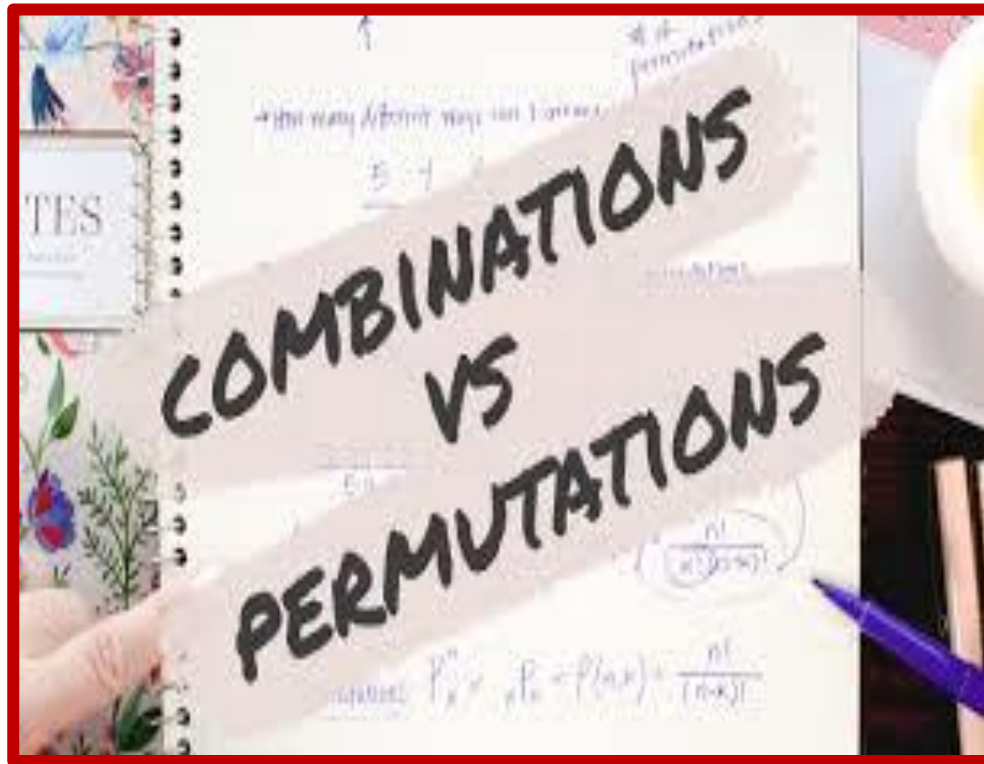




PERMUTATIONS AND COMBINATIONS

Module-10



Confused

Should I unlock with Permutation or Combination?

Permutation

Combination

I AM LOCKED

I AM LOCKED

QUESTIONS.....

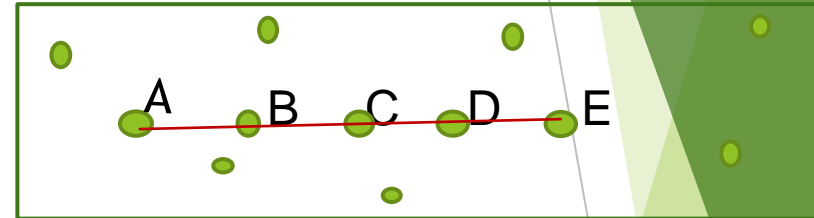
Q6) In how many ways can a committee of 5 persons with a chairperson be selected from 12 persons?

No. of ways of selecting a chairperson from 12 persons = ${}^{12}C_1$ ways.

And the remaining 4 persons can be selected from the remaining 11 in ${}^{11}C_4$ ways.

$$\begin{aligned} \therefore \text{Total no. of ways} &= {}^{12}C_1 \times {}^{11}C_4 \quad \text{And} \Rightarrow \times \\ &= 12 \times \frac{11 \times 10 \times 9 \times 8}{4 \times 3 \times 2 \times 1} \\ &= 12 \times 330 = 3960. \end{aligned}$$

Q7) Out of 12 points in a plane, no three are in the same line except five points which are collinear. Find the no. of lines that can be formed.



No. of lines formed from 12 points taking 2 at a time ${}^{12}C_2$

No. of lines formed from 5 points taking 2 at a time = 5C_2

But 5 collinear points, when joined pairwise, results in only ONE line.

$$\text{Required no. of straight lines} = {}^{12}C_2 - {}^5C_2 + 1$$

QUESTIONS.....

Q 8). If ${}^n C_{r-1} = 36$, ${}^n C_r = 84$ and ${}^n C_{r+1} = 126$, then find the value of ${}^r C_2$.

Sol. We know that $\frac{{}^n C_r}{{}^n C_{r-1}} = \frac{n-r+1}{r}$.

$$\therefore \frac{n-r+1}{r} = \frac{84}{36} \quad (\text{given})$$

$$\Rightarrow \frac{n-r+1}{r} = \frac{7}{3} \quad \Rightarrow 3n - 3r + 3 = 7r$$

$$\Rightarrow 10r - 3n = 3 \quad \text{-----(i)}$$

$$\frac{{}^n C_{r+1}}{{}^n C_r} = \frac{n-(r+1)+1}{r+1} = \frac{126}{84} \quad (\text{given})$$

$$\therefore \frac{n-r}{r+1} = \frac{3}{2} \quad \Rightarrow 2n - 2r = 3r + 3$$

$$\Rightarrow 2n - 5r = 3 \quad \text{-----(ii)}$$

Solving (i) and (ii), we get $n = 9$ and $r = 3$.

$$\therefore {}^r C_2 = {}^3 C_2 = 3$$

Q9) Find the no. of ways in which we can choose a committee from four men and six women, so that the committee includes at least two men and exactly twice as many women as men.

At least 2
2 and more

Men	Women
2	4
3	6

Required no. of committee formed

$$= {}^4 C_2 \times {}^6 C_4 + {}^4 C_3 \times {}^6 C_6$$

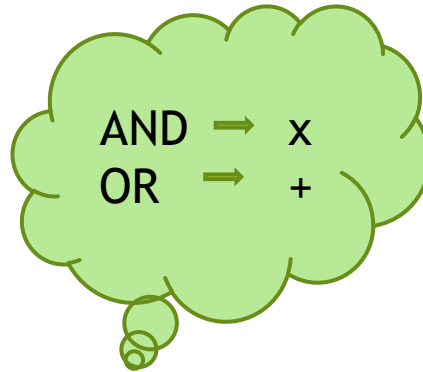
$$= \frac{4 \times 3}{2} \times \frac{6 \times 5}{2} + 4 \times 1$$

$$= 6 \times 15 + 4 = 94$$

QUESTIONS.....

Q 10) In how many ways can we get exactly 4 hearts or exactly 3 spades in a draw of 6 cards?

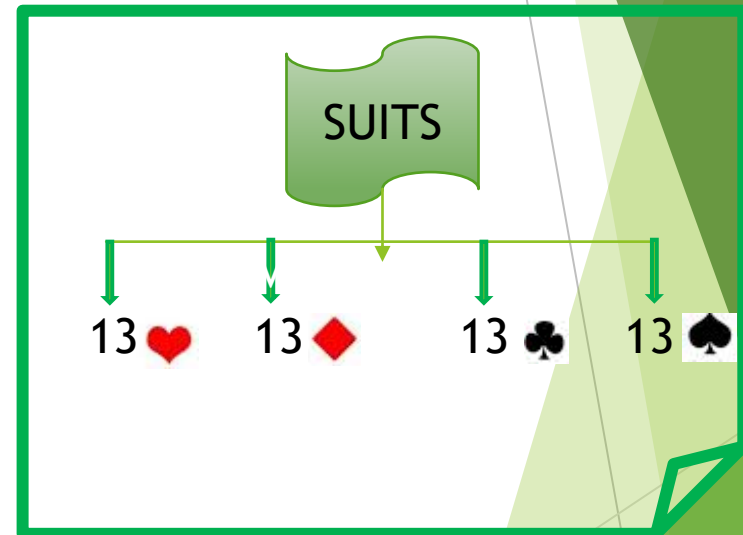
Exactly 4 hearts \longrightarrow ${}^{13}C_4$
Remaining 2 cards \longrightarrow ${}^{39}C_2$



Exactly 3 spades \longrightarrow ${}^{13}C_3$
Remaining 3 cards \longrightarrow ${}^{39}C_3$

\therefore No. of ways of drawing exactly 4 hearts or 3 hearts

$$= {}^{13}C_4 \times {}^{39}C_2 + {}^{13}C_3 \times {}^{39}C_3$$



Determine whether each situation involves permutations or combinations.....

Selection of a committee of 3 from 10 people

Arrangement of 10 books on a shelf

A draw of 6 cards from a deck of 52 cards

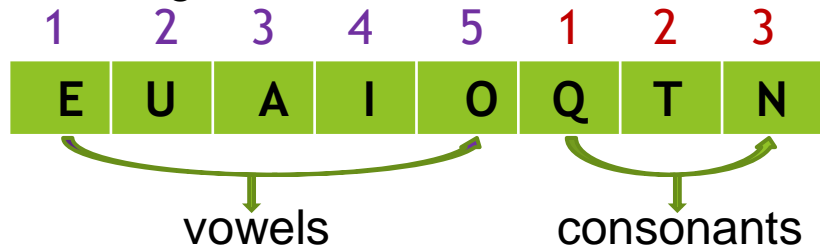
A subset of 12 elements contained in a set of 26

A guest list of 3 friends that your family has said you can invite to dinner

Arrangement of 12 people around a table

MISCELLANEOUS QUESTIONS.....

Q1) How many words with or without meaning can be formed using all the letters of the word 'EQUATION' at a time so that the vowels and consonants occur together?



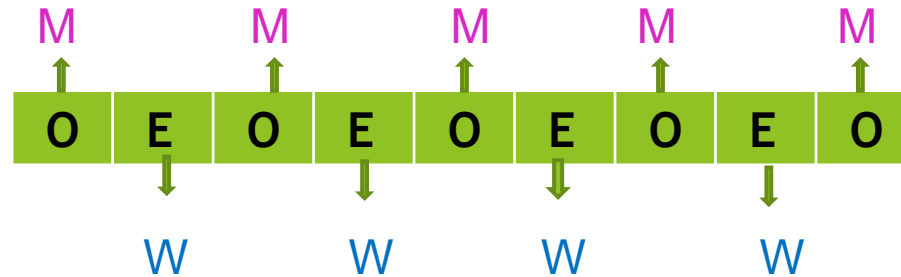
Vowels arranged $\longrightarrow 5!$ ways

Consonants arranged $\longrightarrow 3!$ ways

The 2 groups of vowels and consonants are arranged in $2!$ ways.

\therefore Total no. of words = $5! \times 3! \times 2!$
 $= 120 \times 6 \times 2 = 1440$

Q2) It is required to seat 5 men and 4 women in a row so that the women occupy even places. How many such arrangements are possible?



Arrangement of men $\longrightarrow 5!$ ways

Arrangement of women $\longrightarrow 4!$ ways

\therefore Required no. of arrangements
 $= 5! \times 4! = 120 \times 24 = 2880$

QUESTIONS.....

Q3) How many words with or without meaning each of 2 vowels and 3 consonants can be formed from the letters of the word 'DAUGHTER' ?

The word DAUGHTER has

Vowels	Consonants
A, U, E	D, G, H, T, R

any 2

any 3

Selecting 2 vowels $\rightarrow {}^3C_2$ ways

Selecting 3 consonants $\rightarrow {}^5C_3$ ways

No. of words formed

$$= {}^3C_2 \times {}^5C_3 \times 5!$$

$$= 3 \times \frac{5 \times 4}{2 \times 1} \times 120 = 3600.$$

Selected 5 letters are arranged

Q4) In how many ways can the letters of the word 'ASSASSINATION' be arranged so that all the S's are together?

S S S S A I N A T I A O N
1 2 3 4 5 6 7 8 9 10

Here,

A \rightarrow 3 times

I \rightarrow 2 times

N \rightarrow 2 times

Required no. of words

$$= \frac{10!}{3!2!2!} = 1,51,200.$$

QUESTIONS.....

Q5) A committee of 12 is to be formed from 9 women and 8 men. In how many ways can this be done if at least five women have to be included in a committee? In how many of these committees, (i) the women are in majority? (ii) the men are in majority?

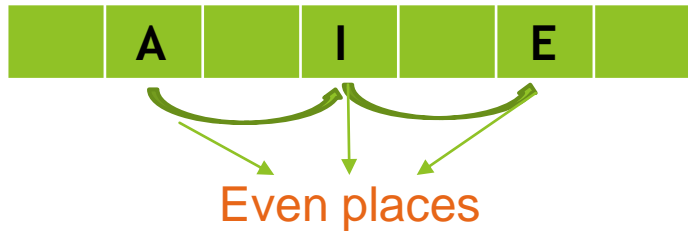
Women	Men	No. of ways
5	7	${}^9C_5 \times {}^8C_7$
6	6	${}^9C_6 \times {}^8C_6$
7	5	${}^9C_7 \times {}^8C_5$
8	4	${}^9C_8 \times {}^8C_4$
9	3	${}^9C_9 \times {}^8C_3$

$$\begin{aligned} \text{No. of committees with at least 5 women} &= ({}^9C_5 \times {}^8C_7) + ({}^9C_6 \times {}^8C_6) + ({}^9C_7 \times {}^8C_5) + \\ &({}^9C_8 \times {}^8C_4) + ({}^9C_9 \times {}^8C_3) \\ &= 1008 + 2352 + 2016 + 630 + 56 = 6062 \end{aligned}$$

- (i) No. of committees where women are in majority = $2016 + 630 + 56 = 2702$
- (ii) No. of committees where men are in majority = 1008

QUESTIONS.....

Q6) Find the number of words which can be formed out of the letters of the word 'ARTICLE', so that the vowels occupy even places.



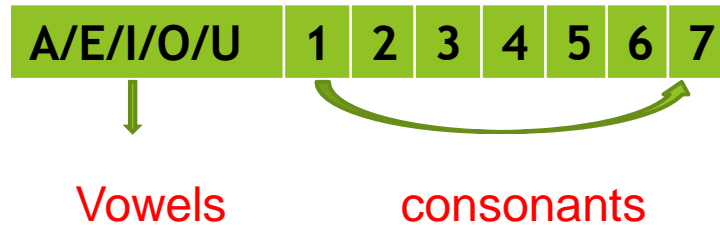
Vowels arranged $\rightarrow 3!$ ways

Consonants arranged $\rightarrow 4!$ ways

\therefore Total no. of words = $3! \times 4!$

$$= 6 \times 24 = 144$$

Q7) Find the number of different 8 - letter words formed from the letters of the word 'EQUATION', if each word is to start with a vowel.



Vowels as the first letter $\rightarrow 5$ ways

Remaining 7 places $\rightarrow 7!$ ways

No. of 8 - letter words formed

$$= 5 \times 7! = 5 \times 5040 = 25,200.$$

HOMEWORK QUESTIONS.....

Q1) The English alphabet has 5 vowels and 21 consonants. How many words with 2 different vowels and 2 different consonants can be formed from the alphabets?

Q2) There are 10 points in a plane of which 4 are collinear. Find the number of straight lines obtained by joining any two of them.

Q3) Out of 8 men and 4 ladies, a committee of 5 is to be formed. In how many ways can this be done so as to include at least one lady?

Q4) How many words, each of 3 vowels and 2 consonants can be formed from the letters of the word INVOLUTE ?

Q5) Find the number of ways in which the letters of the word 'ARRANGE' be arranged such that both R do not come together.

ANSWERS:

Q1) 50400 Q2) 40 Q3) 736 Q4) 2880 Q5) 900